Master of Science Program in Financial Mathematics

The Department of Mathematics (https://math.uchicago.edu/) offers a separate Master of Science in Financial Mathematics degree. Students of the Financial Mathematics Program (http://www-finmath.uchicago.edu/) develop a thorough understanding of the theoretical background of pricing models for financial derivatives and the underlying assumptions. Moreover, students learn to critically ascertain the applicability and limitations of these various models.

The Financial Mathematics Program offers accelerated, integrated coursework that explores the deep-rooted relationship that exists between theoretical and applied mathematics and the ever-evolving world of finance. Professionals from the financial industry instruct a significant number of classes in the program, using various methods to explore how models behave in practice under a variety of market conditions, as well as to evaluate the validity of underlying assumptions and consequential violations of these assumptions. Students will learn to use these models to set up and evaluate the effectiveness of hedges by simulating various market conditions.

Full-time students following the five-quarter track complete the Financial Mathematics curriculum in five quarters, or 15 months. Part-time students, on average, complete the Program in two to three academic years. The Program must be completed within four academic years from the date of matriculation. For the convenience of our working students, classes meet for three hours on weekday evenings (6pm - 9pm) and are video recorded. Students are permitted to take 300 units for a non-quality Pass/Fail grade towards their degree completion.

Various software packages and data providers, (including Bloomberg terminals,) are licensed to the Program and will be provided free of charge.

The Financial Mathematics Program seeks candidates with a solid background in mathematics developed through majors such as mathematics, statistics, engineering, science, and economics. Additionally, relevant work experience and experience with computer programming skills, (including C++ and Python) are strongly taken into consideration by the Admissions Committee. We admit driven individuals that come from diverse educational, social, and geographic backgrounds. Candidates should be able to demonstrate excellence in both academics and leadership.

1250 units are needed to complete the Master of Science in Financial Mathematics:

- Foundations Courses - 350 units (https://finmath.uchicago.edu/curriculum/required-courses/)
- Elective Courses - 900 units (https://finmath.uchicago.edu/curriculum/electives-1/)

Concentrations

Financial Mathematics MS students may qualify for degree concentrations by completing concentration requirements as outlined below. Students are not required to have a concentration. Concentrations will be noted on student transcripts.

The following concentrations are available:

- **Financial Computing**
  250 units from: FINM 31000, 31200, 32600, 32700, 32900, 32950
- **Financial Data Science**
  300 units from: FINM 33160, 33165, 33210, 33220, 34800
- **Options and Derivatives**
  250 units from: FINM 32000, 34500, 34510, 37500
- **Trading and Risk**
  300 units from: FINM 33150, 35000, 35700, 35910, 36702, 37301, 37400, 37601

Project Lab

Project Lab allows students to work with industry partners on their firms' research projects. Students get to apply their coursework in new ways while getting a closer look at the challenges and opportunities in industry research. Project Lab is a 10-week collaboration, and each team is comprised of 4-6 students supported by a faculty advisor. The course (FINM 36000) is typically offered every quarter, and students apply to specific firm projects. The project concludes with a final presentation to the firm and completion of specific project deliverables. Project Lab is a repeatable course, as many students find it a valuable way to build industry experience throughout the program.

Financial Mathematics Courses

FINM 31200. Blockchains and Cryptoassets for Finance. 50 Units.
Since before the invention of reeding for the edges of coins, technology has been closely involved with money. The rise of cryptotokens and associated cryptocurrencies is the latest manifestation of their entanglement. This

class is aimed at understanding crypto from the point of view of finance professionals. We will study some of the basic concepts in crypto, and then quickly move on to the world of smart contracts and emissions, cryptotokens' analogs to derivatives and interest-bearing securities.
Instructor(s): Brian Boonstra, William Cottrell, Christopher Ellison Terms Offered: Autumn. Course meets every other week.
Note(s): Course meets every other week.

FINM 31400. Introduction to Blockchain and Smart Contracts. 100 Units.
This course is a comprehensive technical introduction to relevant topics in the wider ecosystem surrounding blockchain and smart contracts. Our technological focus will include substantive topics in the fundamental problems that blockchains such as Bitcoin and Ethereum are attempting to solve (and are generating), and will focus on implementation details including algorithms, cryptography, security and trust, peer-to-peer networking, distributed ledgers, double spending, consensus algorithms, decentralized applications, smart contracts, and supporting technologies. With that said, this is not a course in economics or monetary theory, trading cryptocurrencies, nor is it a course on regulatory or legal issues surrounding blockchain and smart contracts, although we will touch on many of these topics throughout the course. We will also cover broader applications of blockchain+smart contract technology beyond cryptocurrencies and ICOs and NFTs including use cases from finance, voting, real estate, medicine, digital arts and NFTs (music, art).
Terms Offered: Summer
Prerequisite(s): MPCS 51036 or 51040 or 51042 or 51046 or 51100 or CAPP 30122 or FINM 32500.
Note(s): Non-MPCS students must receive approval from program prior to registering. Request form available online https://mpcs-internal.cs.uchicago.edu.
Equivalent Course(s): MPCS 56605

FINM 32000. Numerical Methods. 100 Units.
Implementing the theory introduced in Mathematical Foundations of Option Pricing (FINM 33000), this course takes a numerical/computational approach to the pricing and hedging of financial derivatives. Topics include: Trees as diffusion approximations; Finite difference methods for PDE solution; Monte Carlo methods for simulation; Fourier transform methods for pricing.
Instructor(s): R. Lee Terms Offered: Spring
Note(s): Computing.

FINM 32500. Computing for Finance in Python. 100 Units.
This course is intended to teach basic programming concepts and techniques to students desiring to work in the financial sector. It is tailored for students without prior programming experience. At the end of this class, students will have the necessary programming skills to be successful in their daily activities. We will cover the basics: control structures, data structures, functions, algorithms, and debugging. Additionally, we will cover object-oriented design and Python specific data handling. We will work on several projects aimed at building a real trading system.
Instructor(s): Sebastien Donadio Terms Offered: Autumn
Note(s): Students must take at least one of: FINM 32500 and FINM 33160 towards the computing requirement.

FINM 32600. Computing for Finance in C++ 100 Units.
No previous programming knowledge is assumed. In Computing for Finance in C++, we will introduce the syntax and semantics of C++ and basics of OO programming. As part of the course work, students will develop an OO option pricer using the Monte Carlo technique. Classes are taught using a combination of lectures and in class hands-on lab sessions.
Instructor(s): C. Liyanaarachchi Terms Offered: Winter
Note(s): Student must take at least one of FINM 32600 or FINM 32700 towards the computing requirement.

FINM 32700. Advanced Computing for Finance. 100 Units.
This course is intended to teach programming concepts and techniques to students desiring to work in the financial sector. At the end of this class, students will have the necessary programming skills to be successful in their daily activities. We will cover the required skills to work as a quantitative researcher: advanced data structures (STL, Boost), parallel programming, inter-process communication, linear algebra computation, simulation and modeling. We will work on several projects aimed at building several trading strategies using C+ +. Classes are taught using a combination of lectures and in class hands-on lab sessions.
Instructor(s): S. Donadio Terms Offered: Spring
Note(s): Student must take at least one of FINM 32600 and FINM 32700 towards computing requirement.

FINM 32900. Full-Stack Quantitative Finance. 100 Units.
Data Science for Finance" is a hands-on course centered on key data science tools in quantitative finance. Acknowledging the field's wide scope, the course focuses on a common skill set across various data science subfields. That is, this course examines elements of the analytical pipeline, from data extraction and cleaning to exploratory analysis, visualization, and modeling, and finally, publication and deployment. It does so with the aim of teaching the tools and principles behind creating reproducible and scalable workflows, including build automation, dependency management, unit testing, the command-line environment, shell scripting, Git for version control, and GitHub for team collaboration. These skills are taught through case studies, each of which will additionally give students practical experience with key financial data sets and sources such as CRSP and Compustat for pricing and financials, macroeconomic data from FRED and the BEA, bond transactions from

FINRA TRACE, Treasury auction data from TreasuryDirect, textual data from EDGAR, and high-frequency trade and quote data from NYSE. Prior experience at an intermediate level with Python and the PyData stack is assumed.
Instructor(s): Jeremy Bejarano Terms Offered: Winter

FINM 32950. Introduction to HPC in Finance. 50 Units.
This short course introduces parallel programming and related concepts using some popular technologies (e.g. Intel's family of parallel models, OpenMP, CUDA etc.) at an introductory level. Application performance improvement using a systematic and structured approach is illustrated. Applications in finance are used to illustrate how to exploit parallelism to solve large scale computing problems. No prior knowledge of parallel computing is assumed. Previous coursework in C++ or Python (FINM 32500 or 32600 or 32700) is required.
Instructor(s): C. Liyanaarachchi Terms Offered: Summer
Note(s): Counts towards elective requirement.

FINM 33000. Option Pricing. 100 Units.
Introduction to the theory of arbitrage-free pricing and hedging of financial derivatives. Topics include: Arbitrage; Fundamental theorems of asset pricing; Binomial and other discrete models; Black-Scholes and other continuous-time Gaussian models in one-dimensional and multidimensional settings; PDE and martingale methods; Change of numeraire. Program requirement.
Instructor(s): Roger Lee Terms Offered: Autumn

FINM 33150. Quantitative Trading Strategies. 100 Units.
Quantitative trading strategies, employing investment decisions based on model output, are a major component of business operations in the finance industry worldwide. We will present the major components of these strategies as found in several asset classes (equities, futures, credit, FX, interest rates and energy). A large proportion of the models involved in quantitative strategies are expressible in terms of regressions. We will cover most of the ways they are used, including practical tricks and considerations, and concentrating particularly on achieving trustworthy performance. Mathematically, we will cover the computation of linear regressions with and without weights, in univariate and multivariate cases, having least squares or other objective functions. Of the major computation technologies actively used by the finance industry (C/C++, Matlab, Java, R, VB/Excel, C\#, Python) we have chosen R and Python for numerical computation, with (very) light usage of Excel and with data coming from Quandl and some proprietary sources. Program requirement.
Instructor(s): B. Boonstra Terms Offered: Winter

FINM 33160. Machine Learning in Finance. 100 Units.
The course will focus on two Machine Learning categorization models: Logistic Regression and Support Vector Machines, both binary and multi-category. The course will develop the mathematical foundations for these models and the optimization algorithms for training them on actual data. The algorithms will be implemented in Python. The necessary parts of Python programming will be taught along the way as they are needed. The Machine Learning models will be used to train models for trading stocks based on both fundamental and technical data. The models will be implemented in Python, using several Machine Learning libraries such as Scikitlearn and back-tested using the web service Quantopian. At the end of the course, the students will develop and implement their own trading models and analyze the performance of their models.
Instructor(s): N. Nygaard Terms Offered: Winter
Prerequisite(s): FINM 32500 or Python waiver.
Note(s): Students must take at least one of FINM 32500 and FINM 33160 towards computing requirement.

FINM 33165. Generative Models. 100 Units.
ChatGPT, DALL-E, Stable Diffusion. These are all examples of Generative AI models. Generative models are a class of machine learning techniques that can learn to synthesize realistic data from a dataset, such as datasets of images, text, or audio, etc. In this course, we will cover the mathematical foundations and practical implementation of generative models. This includes topics such as Probability Distributions, Maximum likelihood estimation, Bayesian inference, Variational inference, Monte Carlo methods and Markov Chain Monte Carlo techniques. We will explore and implement generative models, such as Restricted Boltzmann Machines (RBM), Variational Autoencoders (VAE), Energy Based Models (EBM), Transformers (GPT) and Diffusion Processes (DDPM). The course will consist of nine lectures, four homework assignments, a Midterm and a Final. Each week a discussion topic will be posted on the discussion board. Extra points can be earned by thoughtful contributions to the discussions. Prerequisites: Linear algebra, calculus, probability theory (such as Greg Lawler's course) and basic programming skills in Python.
Instructor(s): N. Nygaard Terms Offered: Autumn
Prerequisite(s): FINM 33160 or FINM 33161/33162 or Consent of Instructor
Note(s): Counts towards computing requirement.

FINM 33170. Financial Statistics: Time Series, Forecasting, Mean Reversion, and High Frequency Data. 100 Units.
This course is an introduction to the econometric analysis of high-frequency financial data. This is where the stochastic models of quantitative finance meet the reality of how the process really evolves. The course is focused on the statistical theory of how to connect the two, but there will also be some data analysis. With some additional statistical background (which can be acquired after the course), the participants will be able to read

articles in the area. The statistical theory is longitudinal, and it thus complements cross-sectional calibration methods (implied volatility, etc.). The course also discusses volatility clustering and market microstructure.
Terms Offered: Winter
Prerequisite(s): Some statistics/econometrics background as in STAT 24400–24500, or FINM 33150 and FINM 33400, or equivalent, or consent of instructor.
Equivalent Course(s): STAT 33910

FINM 33210. Bayesian Statistical Inference and Machine Learning. 50 Units.
The course will develop a general approach to building models of economic and financial processes, with a focus on statistical learning techniques that scale to large data sets. We begin by introducing the key elements of a parametric statistical model: the likelihood, prior, and posterior, and show how to use them to make predictions. We shall also discuss conjugate priors and exponential families, and their applications to big data. We treat linear and generalized-linear models in some detail, including variable selection techniques, penalized regression methods such as the lasso and elastic net, and a fully Bayesian treatment of the linear model. As applications of these techniques, we shall discuss Ross' Arbitrage Pricing Theory (APT), and its applications to risk management and portfolio optimization. As extensions, we will discuss multilevel and hierarchical models, and conditional inference trees and forests. We also treat model-selection methodologies including cross-validation, AIC, and BIC and show how to apply them to all of the financial data sets presented as examples in class. Then we move on to dynamic models for time series including Markov state-space models, as special cases. As we introduce models, we will also introduce solution techniques including the Kalman filter and particle filter, the Viterbi algorithm, Metropolis-Hastings and Gibbs Sampling, and the EM algorithm.
Instructor(s): Gordon Ritter
Equivalent Course(s): STAT 33400

FINM 33220. Bayesian Statistical Inference and Machine Learning II. 50 Units.
This course helps to prepare students for careers in finance, financial engineering, and financial data science, both on the sell and the buy sides. The course teaches cutting-edge tools and methods that drive investment decisions at quantitative trading firms, and, more generally, firms applying machine learning to data science. The course will combine presentations of theory, immediately followed by in-class programming examples using real financial data. Students will subsequently build upon these examples in their homework and projects. This is the second half of a one-quarter course that has been split into two half-quarter units. It can be taken without the first part by students who are especially motivated, or who have had prior experience in machine learning. In the first two lectures, we will focus on support vector machines, gaussian processes, neural networks, and kernel-based methods. The final two lectures will focus on the new, but rapidly-growing area of Reinforcement Learning for finance. There will be four lectures, followed by a final project that will use the methods discussed in class on real financial data.
Instructor(s): Gordon Ritter Terms Offered: Autumn
Prerequisite(s): FINM 33210
Note(s): May count towards computing or elective requirements

FINM 34000. Probability and Stochastic Processes. 50 Units.
This course provides a mathematical introduction to probability and stochastic processes. While the main focus is discrete probability and combinatorial analysis, some continuous probability is discussed. Examples and applications are emphasized over theory.
Instructor(s): Greg Lawler Terms Offered: Winter
Note(s): Foundations

FINM 34500. Stochastic Calculus. 100 Units.
The course starts with a quick introduction to martingales in discrete time, and then Brownian motion and the Ito integral are defined carefully. The main tools of stochastic calculus (Ito's formula, Feynman-Kac formula, Girsanov theorem, etc.) are developed. The treatment includes discussions of simulation and the relationship with partial differential equations. Some applications are given to option pricing, but much more on this is done in other courses. The course ends with an introduction to jump process (Levy processes) and the corresponding integration theory. Program requirement.
Instructor(s): G. Lawler Terms Offered: Winter
Equivalent Course(s): STAT 39000

FINM 34510. Stochastic Calculus I. 50 Units.
The course starts with a quick introduction to martingales in discrete time, and then Brownian motion and the Ito integral are defined carefully. The main tools of stochastic calculus (Ito's formula, Feynman-Kac formula, Girsanov theorem, etc.) are developed. The treatment includes discussions of simulation and the relationship with partial differential equations. Some applications are given to option pricing, but much more on this is done in other courses. The course ends with an introduction to jump process (Levy processes) and the corresponding integration theory.
Terms Offered: Winter
Prerequisite(s): Consent of instructor.
Equivalent Course(s): STAT 39010

FINM 34800. Modern Applied Optimization. 100 Units.
This course assumes no background in optimization. The focus will be on various classical and modern algorithms, with a view towards applications in finance, machine learning, and statistics. In the first half of the course we will go over classical algorithms: univariate optimization and root finding (Newton, secant, regula falsi, etc), unconstrained optimization (steepest descent, Newton, quasi-Newton, Gauss-Newton, Barzilai-Borwein, etc), constrained optimization (penalty, barrier, augmented Lagrangian, active set, etc). In the second half of the course we will cover algorithms that have become popular over the last decade: proximal algorithms, stochastic gradient descent and variants, algorithms that involve moments or momentum or mirror, etc. Applications to machine learning and statistics will include ridge/lasso/logistic regression, support vector machines with hinge/sigmoid loss, optimal experimental designs, maximum entropy, maximum likelihood, Gaussian covariance estimation, feedforward neural networks, etc. Applications in finance will include Markowitz classical portfolio optimization, portfolio optimization with diversification or loss risk constraints, bounding portfolio risks with incomplete covariance information, log optimal investment strategy, etc.
Instructor(s): Lek-Heng Lim Terms Offered: Autumn
Equivalent Course(s): STAT 31001, CAAM 31001

FINM 35000. Topics in Economics. 100 Units.
This course introduces some fundamental macroeconomic theories and empirical studies that are relevant to finance. We will cover money, monetary policy, international finance, commercial and central banking, interest rates in the context of the loanable funds market, and risk premiums in the context of the stochastic discount factor.
Terms Offered: Autumn
Note(s): Elective

FINM 35700. Credit Markets. 50 Units.
The course objective is to introduce students to basic concepts on pricing, trading and portfolio risk management for credit instruments. The focus will be on liquid on cash corporate bonds, credit default swaps, credit indices and ETFs. Various trading strategies will be discussed with market practitioners, for a better insight into the "daily activity" of credit trading desks.
Terms Offered: Spring
Prerequisite(s): FINM 37400: Fixed Income

FINM 35900. Macro Finance. 50 Units.
A significant goal of this course is to give you quantitative work while simultaneously evaluating you on your ability to communicate, present and discuss your conclusions, work and ideas. In addition, to pair with the quant skills you should be honing throughout the program (and in this course), this course seeks to give additional intuition and narrow the gap between economic theory and practical industrial application. Objectives • Understand what macro finance is and the relationship between the macroeconomy and financial markets, and how these give rise to a natural backdrop of investing across asset classes. • Understand how macro and multi-asset investors approach investing differently from investing in a single asset class. • Understand examples of both the approach to building and the optimal execution and implementation of macro/multi-asset trading strategies. • Generate your own macro/multi-asset trading strategies. • Understand it is essential that risk managers understand macroeconomic variables, their cross-asset implications and spillovers, and how doing so drastically improves their management of risk. • Conduct your own risk analysis from the perspective of a macro/multi-asset risk manager.
Instructor(s): Ian Wright Terms Offered: Spring

FINM 35910. Applied Algorithmic Trading. 50 Units.
Applied Algorithmic Trading will introduce the required background knowledge and processes necessary for the design and implementation of algorithmic trading models within the context of industry requirements. The objective of the course is to bring together the numerous disciplines covered in other Financial Mathematics courses, focused on quantitative trading, and combine them into a workable industry level presentation. This course will walk students through the process of generating trading ideas, quantifying the trading process, risk-based modeling concepts, back-testing and optimization techniques, and key industry metrics used to evaluate algorithmic trading model performance. Lastly, the course will stress the leadership and presentation skills necessary to make a successful pitch in an industry setting. Program elective.
Instructor(s): Chris Gersch Terms Offered: Autumn
Prerequisite(s): FINM 33150 or consent of instructor.

FINM 36000. Project Lab. 50 Units.
Program elective.
Instructor(s): R. Lee Terms Offered: Autumn Spring Summer Winter
Prerequisite(s): Consent of instructor.

FINM 36001. Project Lab 2. 000 Units.
Program elective.
Instructor(s): R. Lee Terms Offered: Autumn Spring Summer Winter
Prerequisite(s): FINM 36000 and consent of instructor.

FINM 36700. Portfolio and Risk Management. 100 Units.
The course begins by covering the classic foundations of portfolio theory, including mean-variance mathematics and the standard equity factor models used in attribution and risk management. It goes beyond these classic results to cover return dynamics, statistical uncertainty, model selection, market frictions, and non-convex optimization. Throughout, the course examines issues of application and implementation relevant for professionals in various areas of quantitative finance. Case studies cover a range of asset classes, investment strategies, and industries.
Instructor(s): Mark Hendricks Terms Offered: Autumn
Note(s): Program requirement.

FINM 36702. Portfolio Credit Risk: Modeling and Estimation. 50 Units.
The global credit crisis of 2008 taught that credit loss can destroy financial institutions that had previously seemed secure. Students in Portfolio Credit Risk learn the models used to analyze this risk, to limit positions in credit-sensitive instruments, to allocate holding costs to align with risk, and to determine required minimum bank capital. Beyond these specific applications, the modeling of portfolio credit risk provides tools and insights that can be applied when an available data set is sparse relative to the richness of possible outcomes.
Instructor(s): Jon Frye Terms Offered: Spring
Note(s): Either FINM 35500 or FINM 36702 is required. This is a five-week course taught in the first-half of the quarter.

FINM 37301. Foreign Exchange: Markets, Pricing and Products. 50 Units.
This course will examine international currency markets, financial products, and applications of quantitative models with an emphasis on the quantitative methods and derivative products in common use today. Topics will include a) pricing for FX products in theory and in practice, specifically spot, forward, futures, deposits, cross-currency swaps, non-deliverable contracts, and FX options, b) FX markets in practice, exchange rate regimes, international monetary systems, FX modeling and forecasting, and c) practical market applications of FX options, exotic options, and hybrid products.
Instructor(s): Anthony Capozzoli Terms Offered: Winter
Note(s): Counts towards elective requirement. This is a five-week course taught in the second-half of the quarter.

FINM 37400. Fixed Income. 50 Units.
This course covers the mathematical modeling, statistical analysis, and market features relevant for pricing and managing fixed income products. Key topics include the following: the yield curve; interest-rate risk via duration, convexity, and factor approaches; fixed-income products such as swaps, caps, and floors; trading strategies for rates, including inflation and the Expectations Hypothesis. This course does not cover specialized models for analyzing fixed-income derivatives nor the issue of credit-risk. But it covers many fundamentals needed for those courses. The course uses weekly homework assignments consisting of applied problems that use real data and consider actual cases of fixed income risk, pricing, and trading. The course also has a final exam.
Instructor(s): Mark Hendricks Terms Offered: Winter

FINM 37500. Fixed Income Derivatives. 50 Units.
Fixed Income Derivatives will cover the mathematical pricing of fixed income derivatives, as well as their use in practice. Applications will include trading and hedging.
Instructor(s): Mark Hendricks Terms Offered: Spring

FINM 37601. Mathematical Market Microstructure: An Optimization Approach. 50 Units.
Mathematical Market Microstructure: An Optimization Approach for Dynamic Inventory Management and Market Maker Quoting. This course is an introduction to mathematical theory of market microstructure, with key applications in solving optimal execution problems with inventory management. We will start from discussions of market design, global market structure, algorithmic trading and market making practices. We will then present traditional market microstructure theory in the context of dealer inventory management and information-based quoting and pricing. Latest literature about realized volatility calculations and intraday implied volatility surface modeling using high-frequency data will be reviewed. The subject of order book dynamics research with applications to market impact modeling will be discussed as well. Finally, a review on continuous-time stochastic control theory will be provided and a discussion will be given on execution algorithm development and market making strategy design using stochastic programming techniques. The main goal of this course is to provide a clear discussion on key mathematical treatments and their practical applications of market microstructure problems, in particular relating to price discovery and utility optimization for certain transaction processes with non-trivial transaction cost present. Program elective.
Instructor(s): H. Chou Terms Offered: Autumn
Note(s): This is a five-week course taught in the first half of the quarter.

FINM 38000. Financial Mathematics Practicum. 000 Units.
Program elective.
Instructor(s): Roger Lee Terms Offered: Autumn Spring Summer Winter

FINM 38001. Financial Mathematics Practicum II. 50 Units.
Program elective.
Instructor(s): Roger Lee Terms Offered: Autumn Spring Summer Winter

FINM 38500. Career Seminar. 000 Units.
Presentations/workshops/networking events related to career development in quantitative finance. Program requirement.
Instructor(s): Career Development Office Terms Offered: Autumn Spring Winter