Department of Mathematics

Chair

• Kevin Corlette

Professors

• Laszlo Babai, Computer Science and Mathematics
• Guillaume Bal, Statistics and Mathematics
• Alexander A. Beilinson
• Danny Calegari
• Francesco Calegari
• Kevin Corlette
• Jack D. Cowan
• Marianna Csörnyei
• Vladimir Drinfeld
• Todd Dupont, Computer Science and Mathematics
• Matthew Emerton
• Alex Eskin
• Benson Farb
• Robert A. Fefferman
• Victor Ginzburg
• Denis Hirschfeldt
• Kazuya Kato
• Carlos E. Kenig
• Gregory Lawler, Mathematics and Statistics
• J. Peter May
• Andre Neves
• Bao Châu Ngô
• Madhav Vithal Nori
• Alexander Razborov, Mathematics and Computer Science
• Luis Silvestre
• Charles Smart
• Panagiotis Souganidis
• Sidney Webster
• Shmuel Weinberger
• Amie Wilkinson
• Robert Zimmer

Associate Professors

• Simion Filip
• Roger Lee
• Maryanthe Malliaris

Assistant Professors

• Sebastian Hurtado-Salazar
• Dana Mendelson
• Nikita Rozenblyum
• Daniil Rudenko

Instructors

• Lucas Benigni
• Guher Camliyurt
• Elliot Cartee
• Mark Cerenzia
The Department of Mathematics (http://www.math.uchicago.edu/) provides a comprehensive education in mathematics which takes place in a stimulating environment of intensive research activity. The graduate
program includes both pure and applied areas of mathematics. Ten to fifteen graduate courses are offered every quarter. Several seminars take place every afternoon. There is an active visitors program with mathematicians from around the world coming for periods from a few days to a few months. There are four major lecture series each year: the Adrian Albert Lectures in Algebra, the Antoni Zygmund and Alberto Calderón Lectures in Analysis, the Unni Namboodiri Lectures in Topology, and the Charles Amick Lectures in Applied Mathematics. The activities of the department take place in Eckhart and Ryerson Halls. The Departments of Mathematics, Computer Science and Statistics have several joint appointments, and they coordinate their activities.

**Graduate Degrees in Mathematics**

The graduate program of the Department of Mathematics is oriented towards students who intend to earn a Ph.D. in mathematics on the basis of work done in mathematics. The Department also offers the degree of Master of Science in mathematics, which is acquired as the student proceeds on to the Ph.D. degree. Students are not admitted with the Master of Science degree as their final objective. In addition, the department offers a separate Master of Science in Financial Mathematics degree program which is taught in the evenings. See the program listing for Financial Mathematics ([http://collegecatalog.uchicago.edu/graduate/departmentofmathematics/financialmathematics/](http://collegecatalog.uchicago.edu/graduate/departmentofmathematics/financialmathematics/)) for more information.

The divisional requirements for these degrees can be found in the section on the Physical Sciences Division in these Announcements. Otherwise, the requirements are as follows.

**The Degree of Master of Science**

The candidate must pass, the nine basic first year graduate courses in the areas of

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<tr>
<th>Course</th>
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<tr>
<td>MATH 32500</td>
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<td>MATH 31800</td>
<td>Topology and Geometry II</td>
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<td>MATH 31900</td>
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At the beginning of each quarter a placement exam is offered for each of the courses above. Students who pass the exam can place out of the course, but must take another course in a related area.

**The Degree of Doctor of Philosophy**

For admission to candidacy for the Doctor of Philosophy, an applicant must demonstrate the ability to meet both the divisional requirements and the departmental requirements for admission.

The applicant must satisfy the above mentioned requirements for the degree of Master of Science in mathematics.

The applicant must satisfactorily complete a topic exam. This exam covers material that is chosen by the student in consultation with members of the department and is studied independently. The topic presentation is normally made by the end of the student's second year of graduate study, and includes both a written proposal and an oral presentation and exam.

The applicant must also successfully complete the department's program of preparatory training in the effective teaching of mathematics in the English language at a level commensurate with the level of instruction at the University of Chicago.

After successful completion of the topic presentations, the student is expected to begin research towards the dissertation under the guidance of a member of the department. The remaining requirements are to:

1. Complete a dissertation containing original, substantial, and publishable mathematical results
2. Present the contents of the dissertation in an open lecture
3. Pass an oral examination based both on the dissertation and the field of mathematics in which it lies

A joint Ph.D. in Mathematics and Computer Science is also offered. To be admitted to the joint program, students must be admitted by both departments as follows. Each student in this program will have a primary program (either Math or CS). Students apply to their primary program. Once admitted, they can apply to the secondary program for admission to the joint program. This secondary application can occur either before they
enter the program or any time during their first four years in their primary program. Simultaneous applications to both programs will also be considered (one of the programs being designated as primary).

Students enrolling in this program need to satisfy the course requirements of both departments. They have to satisfy the course requirements of their primary program on the schedule of that program, and satisfy the course requirements of their secondary program by the end of their fifth year. They also need to satisfy the examination requirements of their primary program, and are expected to write a dissertation in an area relevant to both fields.

**MATH 30200-30300-30400. Computability Theory-I; Computability Theory-II; Computability Theory-III.**

The courses in this sequence are offered in alternate years.

**MATH 30200. Computability Theory I. 100 Units.**

We investigate the computability and relative computability of functions and sets. Topics include mathematical models for computations, basic results such as the recursion theorem, computably enumerable sets, and priority methods.

Instructor(s): D. Hirschfeldt Terms Offered: Spring
Prerequisite(s): Consent of department counselor. MATH 25500 or consent of instructor.
Equivalent Course(s): CMSC 38000

**MATH 30300. Computability Theory II. 100 Units.**

CMSC 38100 treats classification of sets by the degree of information they encode, algebraic structure and degrees of recursively enumerable sets, advanced priority methods, and generalized recursion theory.

Instructor(s): D. Hirschfeldt Terms Offered: Spring
Prerequisite(s): Consent of department counselor. MATH 25500 or consent of instructor.
Equivalent Course(s): CMSC 38100

**MATH 30400. Computability Theory-III. 100 Units.**

**MATH 30708. Simple Theories. 100 Units.**

Simple theories (so called), introduced almost forty years ago, provide a model theoretic framework for studying certain families of 'random' objects, such as the theories of random graphs and hypergraphs. Very recent work has shown the class to contain a much greater range of complexity than previously thought. This course will cover the fundamental theorems of simple theories along with some of the new developments.

Instructor(s): Maryanthe Malliaris Terms Offered: Autumn

**MATH 30813. Some Logic and Geometry for Mathematicians. 100 Units.**

This quarter course will cover three major applications of logic to other branches of mathematics. The first is Tarski's theorem about quantifier elimination and decidability of the first order theory of the reals. This is a cornerstone of real algebraic geometry with major implications in analysis and geometry. The second is the existence of groups with unsolvable word problem and Higman's characterization of finitely generated subgroups of finitely generated groups. This implies that some geometric problems, such as the homeomorphism problem for manifolds, or deciding whether a simplicial complex is a manifold are undecidable (as we hope to explain). And, finally, we will explain why there is no algorithm to decide whether Diophantine equations with integer coefficients have integral solutions, along with some interesting definability questions in this context.

Instructor(s): Shmuel Weinberger, Maryanthe Malliaris Terms Offered: Spring

**MATH 30900-31000. Model Theory I-II.**

MATH 30900 covers completeness and compactness; elimination of quantifiers; omission of types; elementary chains and homogeneous models; two cardinal theorems by Vaught, Chang, and Keisler; categories and functors; inverse systems of compact Hausdorff spaces; and applications of model theory to algebra. In MATH 31000, we study saturated models; categoricity in power; the Cantor-Bendixson and Morley derivatives; the Morley theorem and the Baldwin-Lachlan theorem on categoricity; rank in model theory; uniqueness of prime models and existence of saturated models; indiscernibles; ultraproducts; and differential fields of characteristic zero.

**MATH 30900. Model Theory I. 100 Units.**

First graduate course in model theory, covering the basics of the modern field, through stability.

Prerequisite(s): MATH 25500 or 25800
Note(s): This course is offered in alternate years.

**MATH 31000. Model Theory II. 100 Units.**

Second graduate course in model theory, focusing on the fundamentals of classification theory.

Terms Offered: Spring
Prerequisite(s): MATH 30900
Note(s): This course is offered in alternate years.

**MATH 31200-31300-31400. Analysis I-II-III.**

Analysis I-II-III
MATH 31200. Analysis I. 100 Units.
Topics include: Lebesgue integration, Lp spaces and Banach spaces, differentiation theory, Hilbert spaces and Fourier series, probability spaces and random variables, strong law of large numbers, central limit theorem, conditional expectation and martingales, Brownian motion.
Terms Offered: Autumn
Prerequisite(s): MATH 26200, 27000, 27200, and 27400; and consent of director or co-director of undergraduate studies

MATH 31300. Analysis II. 100 Units.
Topics include: Hilbert spaces, projections, bounded and compact operators, spectral theorem for compact selfadjoint operators, unbounded selfadjoint operators, Cayley transform, Banach spaces, Schauder bases, Hahn-Banach theorem and its geometric meaning, uniform boundedness principle, open mapping theorem, Frechet spaces, applications to elliptic partial differential equations, Fredholm alternative.
Terms Offered: Winter
Prerequisite(s): MATH 31200

MATH 31400. Analysis III. 100 Units.
Topics include: Basic complex analysis, Cauchy theorem in the homological formulation, residues, meromorphic functions, Mittag-Leffler theorem, Gamma and Zeta functions, analytic continuation, monodromy theorem, the concept of a Riemann surface, meromorphic differentials, divisors, Riemann-Roch theorem, compact Riemann surfaces, uniformization theorem, Green functions, hyperbolic spaces, covering spaces, quotients.
Terms Offered: Spring
Prerequisite(s): MATH 31300

MATH 31700-31800-31900. Topology and Geometry I-II-III.

- **MATH 31700. Topology and Geometry I. 100 Units.**
  Topics include: Fundamental group, covering space theory and Van Kampen’s theorem (with a discussion of free and amalgamated products of groups), homology theory (singular, simplicial, cellular), cohomology theory, Mayer-Vietoris cup products, Poincare Duality, Lefschetz fixed-point theorem, some homological algebra (including the Kunneth and universal coefficient theorems), higher homotopy groups, Whitehead’s theorem, exact sequence of a fibration, obstruction theory, Hurewicz isomorphism theorem.
  Terms Offered: Autumn
  Prerequisite(s): MATH 26200, 27000, 27200, and 27400; and consent of director or co-director of undergraduate studies

- **MATH 31800. Topology and Geometry II. 100 Units.**
  Topics include: Definition of manifolds, tangent and cotangent bundles, vector bundles. Inverse and implicit function theorems, Sard’s theorem and the Whitney embedding theorem. Degree of maps. Vector fields and flows, transversality, and intersection theory. Frobenius’ theorem, differential forms and the associated formalism of pullback, wedge product, integration, etc. Cohomology via differential forms, and the de Rham theorem. Further topics may include: compact Lie groups and their representations, Morse theory, cobordism, and differentiable structures on the sphere.
  Terms Offered: Winter
  Prerequisite(s): MATH 31700

- **MATH 31900. Topology and Geometry III. 100 Units.**
  Topics include: Riemannian metrics, connections and curvature on vector bundles, the Levi-Civita connection, and the multiple interpretations of curvature. Geodesics and the associated variational formalism (formulas for the 1st and 2nd variation of length), the exponential map, completeness, and the influence of curvature on the topological structure of a manifold (positive versus negative curvature). Lie groups. The Chern-Weil description of characteristic classes, the Gauss-Bonnet theorem, and possibly the Hodge Theorem.
  Terms Offered: Winter
  Prerequisite(s): MATH 31800

MATH 32500-32600-32700. Algebra I-II-III.

- **MATH 32500. Algebra I. 100 Units.**
  Topics include: Representation theory of finite groups, including symmetric groups and finite groups of Lie type; group rings; Schur functors; induced representations and Frobenius reciprocity; representation theory of Lie groups and Lie algebras, highest weight theory, Schur-Weyl duality; applications of representation theory in various parts of mathematics.
  Terms Offered: Autumn
  Prerequisite(s): MATH 25700-25800-25900, and consent of director or co-director of undergraduate studies
MATH 32600. Algebra II. 100 Units.
This course will explain the dictionary between commutative algebra and algebraic geometry. Topics will include the following. Commutative ring theory; Noetherian property; Hilbert Basis Theorem; localization and local rings; etc. Algebraic geometry: affine and projective varieties, ring of regular functions, local rings at points, function fields, dimension theory, curves, higher-dimensional varieties.
Terms Offered: Winter
Prerequisite(s): MATH 32500

MATH 32700. Algebra III. 100 Units.
According to the inclinations of the instructor, this course may cover: algebraic number theory; homological algebra; further topics in algebraic geometry and/or representation theory.
Terms Offered: Spring
Prerequisite(s): MATH 32600

MATH 34100. Geometric Literacy-1. 100 Units.
This ongoing course might be subtitled: “what every good geometer should know”. The topics will intersperse more elementary background with topics close to current research, and should be understandable to second year students. The individual modules (2-5 weeks each) might be logically interrelated, but we will try to maintain a “modular structure” so that people who are willing to assume certain results as “black boxes” will be able to follow more advanced modules before formally learning all the prerequisites. This years topics might include: basics of symplectic geometry, harmonic maps in geometry, pseudo-Anosov homeomorphisms and Thurston's compactification of Teichmuller space, algebraic geometry for non-algebraic geometers. Prereq: First year graduate sequence.
Instructor(s): Benson Farb Terms Offered: Autumn
Prerequisite(s): First year graduate sequence.

MATH 34200. Geometric Literacy-2. 100 Units.
This ongoing course might be subtitled: “what every good geometer should know”. The topics will intersperse more elementary background with topics close to current research, and should be understandable to second year students. The individual modules (2-5 weeks each) might be logically interrelated, but we will try to maintain a “modular structure” so that people who are willing to assume certain results as “black boxes” will be able to follow more advanced modules before formally learning all the prerequisites. This years topics might include: basics of symplectic geometry, harmonic maps in geometry, pseudo-Anosov homeomorphisms and Thurston's compactification of Teichmuller space, algebraic geometry for non-algebraic geometers. Prereq: First year graduate sequence.
Instructor(s): Benson Farb Terms Offered: Spring
Prerequisite(s): First year graduate sequence.

MATH 34300. Geometric Literacy - 3. 100 Units.
This ongoing course might be subtitled: “what every good geometer should know”. The topics will intersperse more elementary background with topics close to current research, and should be understandable to second year students. The individual modules (2-5 weeks each) might be logically interrelated, but we will try to maintain a “modular structure” so that people who are willing to assume certain results as “black boxes” will be able to follow more advanced modules before formally learning all the prerequisites. This years topics might include: basics of symplectic geometry, harmonic maps in geometry, pseudo-Anosov homeomorphisms and Thurston's compactification of Teichmuller space, algebraic geometry for non-algebraic geometers. Prereq: First year graduate sequence.
Instructor(s): Benson Farb Terms Offered: Spring
Prerequisite(s): First year graduate sequence.

MATH 34500. Topics in Geometry and Topology. 100 Units.
This course will cover various topics ranging from algebraic and differential geometry to algebraic and geometric topology, often with connections to representation theory and number theory. Recent topics have included Hodge theory, Mostow Rigidity, Topology and geometry of K3 surfaces (joint with Eduard Looijenga), "What all the 3-manifolds are", 4-manifold theory: from Seiberg-Witten to the classification of algebraic surfaces (joint with Danny Calegari), and the cohomology of arithmetic groups (joint with Matt Emerton).
Instructor(s): Benson Farb Terms Offered: Winter
Prerequisite(s): The first year math graduate courses or permission of instructor.

MATH 34600. Topics in Geometry and Topology-2. 100 Units.
This course will cover various topics ranging from algebraic and differential geometry to algebraic and geometric topology, often with connections to representation theory and number theory. Recent topics have included Hodge theory, Mostow Rigidity, Topology and geometry of K3 surfaces (joint with Eduard Looijenga), "What all the 3-manifolds are", 4-manifold theory: from Seiberg-Witten to the classification of algebraic surfaces (joint with Danny Calegari), and the cohomology of arithmetic groups (joint with Matt Emerton).
Instructor(s): Benson Farb Terms Offered: Spring
Prerequisite(s): The first year math graduate courses or permission of instructor.

MATH 36000. Proseminar: Topology. 100 Units.
This informal proseminar is devoted to topics in algebraic topology and neighboring fields. Talks are given by graduate students, postdocs, and senior faculty. They range from basic background through current research.
Instructor(s): Staff
MATH 36100. Topology Proseminar. 100 Units.
This informal “proseminar” is devoted to topics in algebraic topology and neighboring fields. Talks are given by graduate students, postdocs, and senior faculty. They range from basic background through current research.
Instructor(s): J. Peter May Terms Offered: Winter

MATH 36200. Topology Proseminar. 100 Units.
The Spring proseminar is a more formal version of the Fall and Winter topology proseminar. It will be taught primarily or completely by May, on topics of interest to the participants.
Instructor(s): J. Peter May Terms Offered: Spring

MATH 36704. Dynamics and Applications. 100 Units.
The course will provide an introduction to basic results and techniques in dynamical systems and then discuss selected applications.
Instructor(s): Simion Filip Terms Offered: Winter

MATH 36888. Pseudodifferential Operators with Applications. 100 Units.
In this course I will introduce classical pseudodifferential operators and their calculus and give some applications, including to variable coefficient Schrodinger equations.
Instructor(s): Carlos Kenig Terms Offered: Autumn
Prerequisite(s): The first year graduate sequence in analysis.

MATH 36918. Min-max Methods in Minimal Surfaces. 100 Units.
Min-max methods in minimal surfaces have produced a series of spectacular results lately and settle old questions. I will develop the Algren-Pitts min-max theory from the beginning and explain how that can be used to prove existence of minimal surfaces.
Instructor(s): Andre Neves Terms Offered: Spring

MATH 37104. Parabolic Equations with Irregular Data and Related Issues. 100 Units.
Instructor(s): Claude Le Bris Terms Offered: Winter

MATH 37105. Topics in Geometric Measure Theory I. 100 Units.
A measure is a way to assign a size to collections of points. Lebesgue measure is the most important example but, depending upon the application, the ‘size’ of a set may be measured in many different, very interesting ways. The interplay between measure and geometry can be extremely subtle and has given rise to powerful ideas that are used in energy minimisation problems, the theory of partial differential equations and the study of fractal geometry. This is an advanced course on geometric measure theory and its applications.
Instructor(s): Marianna Csornyei Terms Offered: Autumn

MATH 37111. Quiver Varieties. 100 Units.
Study of quiver varieties.
Instructor(s): Victor Ginzburg Terms Offered: Spring

MATH 37219. Crystalline Differential Operators. 100 Units.
Introduction to crystalline differential operators.
Instructor(s): Victor Ginzburg Terms Offered: Winter

MATH 38595. Topics in Complex Dynamics. 100 Units.
An introduction to the theory of complex dynamics in 1 dimension, especially the theory of rational maps and rational correspondences. Foundations of the theory, quasiconformal analysis, no wandering domains, Mandelbrot set and variations.
Instructor(s): Danny Calegari Terms Offered: Spring

MATH 38599. Introduction to Floer Theories. 100 Units.
An introduction to the use of gauge theoretic methods in 3-manifold topology, including Seiberg-Witten and Heegaard Floer Homology, connections to taut foliations and sutured manifolds. Thurston norm, contact structures, etc.
Instructor(s): Danny Calegari Terms Offered: Winter

MATH 42002. P-adic Hodge Theory. 100 Units.
Basic things in p-adic Hodge theory are explained.
Instructor(s): Kazuya Kato Terms Offered: Winter
Prerequisite(s): Algebra 1, 2, 3

MATH 47000. Geometric Langlands Seminar. 100 Units.
This seminar is devoted not only to the Geometric Langlands theory but also to related subjects (including topics in algebraic geometry, algebra and representation theory). We will try to learn some modern homological
algebra (Kontsevich’s A-infinity categories) and some “forgotten” parts of D-module theory (e.g. the microlocal approach).
Instructor(s): Alexander Beilinson, Vladimir Drinfeld Terms Offered: Autumn

**MATH 47100. Geometric Langlands Seminar. 100 Units.**
The seminar is devoted to the Geometric Langlands theory and related subjects, which covers topics in algebraic geometry, algebra, and representation theory.
Instructor(s): Alexander Beilinson, Vladimir Drinfeld Terms Offered: Winter

**MATH 47200. Geometric Langlands Seminar. 100 Units.**
The seminar is devoted to the Geometric Langlands theory and related subjects, which covers topics in algebraic geometry, algebra, and representation theory.
Instructor(s): Alexander Beilinson, Vladimir Drinfeld Terms Offered: Spring

**MATH 59900. Reading/Research: Mathematics. 300.00 Units.**
Readings and Research for working on their PhD

**MATH 70000. Advanced Study: Mathematics. 300.00 Units.**
Advanced Study: Mathematics