Department of Mathematics

Chair

- Shmuel Weinberger

Professors

- Jonathan L. Alperin
- Laszlo Babai, Computer Science
- Alexander A. Beilinson
- Danny Calegari
- Francesco Calegari
- Kevin D. Corlette
- Jack D. Cowan
- Marianna Csörnyei
- Vladimir Drinfeld
- Todd Dupont, Computer Science
- Matthew Emerton
- Alex Eskin
- Benson Farb
- Robert A. Fefferman
- Victor Ginzburg
- Denis Hirschfeldt
- Kazuya Kato
- Carlos E. Kenig
- Steven Lalley, Statistics
- Gregory Lawler
- J. Peter May
- Bao Chau Ngo
- Madhav Vithal Nori
- Alexander Razborov, Computer Science
- Wilhelm Schlag
- L. Ridgway Scott, Computer Science
- Panagiotis Souganidis
- Sidney Webster
- Shmuel Weinberger
- Amie Wilkinson
- Robert Zimmer

Associate Professors
• Roger Lee
• Luis Silvestre
• Charles Smart

Assistant Professors
• Jian Ding, Statistics
• Keerthi Madapusi
• Maryanthe Malliaris
• Nikita Rozenblyum
• Mircea Voda

Instructors
• David Aulicino
• David Bate
• Agnes Beaudry
• George Boxer
• Aaron Brown
• Gregory Chambers
• David Cohen
• Matthew Creek
• Dominic Dotterer
• William Feldman
• Ilya Grigoriev
• Boaz Haberman
• Jonathan Hickman
• Sebastian Hurtado-Salazar
• Hao Jia
• Wenjia Jing
• Tianling Jin
• Michael Khanevsky
• Ian Le
• Bao Viet Le Hung
• Brandon Levin
• Kathryn Lindsey
• Sean Li
• Rohit Nagpal
• Jack Shotton
• Aaron Silberstein
• Stanley Snelson
• Andrei Tarfulea
• Jesse Wolfson
The Department of Mathematics (http://www.math.uchicago.edu) provides a comprehensive education in mathematics which takes place in a stimulating environment of intensive research activity. The graduate program includes both pure and applied areas of mathematics. Ten to fifteen graduate courses are offered every quarter. Several seminars take place every afternoon. There is an active visitors program with mathematicians from around the world coming for periods from a few days to a few months. There are four major lecture series each year: the Adrian Albert Lectures in Algebra, the Antoni Zygmund and Alberto Calderón Lectures in Analysis, the Unni Namboodiri Lectures in Topology, and the Charles Amick Lectures in Applied Mathematics. The activities of the department take place in Eckhart and Ryerson Halls. These contiguous buildings are shared with the Departments of Statistics and Computer Science. The Department of Mathematics and the Department of Computer Science have several joint appointments, and they coordinate their activities. The Department of Mathematics also has joint appointments and joint activity with the Department of Physics.

GRADUATE DEGREES IN MATHEMATICS

The graduate program of the Department of Mathematics is oriented towards students who intend to earn a Ph.D. in mathematics on the basis of work done in either pure or applied mathematics. The department also offers the degree of Master of Science in mathematics, which is acquired as the student proceeds on to the Ph.D. degree. Students are not admitted with the Master of Science degree as their final objective. In addition, the department offers a separate Master of Science in Financial Mathematics degree program which is taught in the evenings. See the program listing for Financial Mathematics (collegecatalog.uchicago.edu/graduate/departmentofmathematics/financialmathematics) for more information.

The divisional requirements for these degrees can be found in the section on the Division of the Physical Sciences in these Announcements. The departmental
requirements for students choosing the program in applied mathematics are described below under the heading, Graduate Degrees in Applied Mathematics. Otherwise, the requirements are as follows.

THE DEGREE OF MASTER OF SCIENCE

The candidate must pass, to the instructor’s satisfaction, the nine basic first year graduate courses in the areas of

Algebra

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<th>Course</th>
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<tbody>
<tr>
<td>MATH 32500</td>
<td>Algebra I</td>
<td>100</td>
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<tr>
<td>MATH 32600</td>
<td>Algebra II</td>
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<tr>
<td>MATH 32700</td>
<td>Algebra III</td>
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Analysis

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<tbody>
<tr>
<td>MATH 31200</td>
<td>Analysis I</td>
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<tr>
<td>MATH 31300</td>
<td>Analysis II</td>
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Topology

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<tr>
<td>MATH 31700</td>
<td>Topology and Geometry I</td>
<td>100</td>
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<tr>
<td>MATH 31800</td>
<td>Topology and Geometry II</td>
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<tr>
<td>MATH 31900</td>
<td>Topology and Geometry III</td>
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With the approval of the department, the exceptionally well prepared student may place out of one or more of these courses, and substitute a more advanced course.

If any of these courses are not passed to the instructor’s satisfaction, the student will be required to take an oral exam in those subject areas before receiving the Master of Science degree.

THE DEGREE OF DOCTOR OF PHILOSOPHY

For admission to candidacy for the Doctor of Philosophy, an applicant must demonstrate the ability to meet both the divisional requirements and the departmental requirements for admission.

The applicant must satisfy the above mentioned requirements for the degree of Master of Science in mathematics.

The applicant must satisfactorily complete an oral topic presentation. This presentation covers material that is chosen by the student in consultation with members of the department and is studied independently. The topic presentation is normally made by the end of the student’s second year of graduate study.

The applicant must also successfully complete the department’s program of preparatory training in the effective teaching of mathematics in the English language at a level commensurate with the level of instruction at the University of Chicago.

After successful completion of the topic presentations, the student is expected to begin research towards the dissertation under the guidance of a member of the department. The remaining requirements are to:
1. Complete a dissertation containing original, substantial, and publishable mathematical results
2. Present the contents of the dissertation in an open lecture
3. Pass an oral examination based both on the dissertation and the field of mathematics in which it lies

GRADUATE DEGREES IN APPLIED MATHEMATICS

The Department of Mathematics, through the Computational and Applied Mathematics Program (CAMP), offers interdisciplinary programs in applied mathematics leading to S.M. and Ph.D. degrees. These programs overlap with but are different from the program in pure mathematics and allow for variations depending on the direction of applications the student chooses. Students choosing the applied mathematics program will participate in courses and seminars not only with pure mathematics students, but also with students in the sciences who have chosen an applied mathematics emphasis in their own departments.

Expanded activity in applied mathematics is occurring within the Department of Mathematics and in the Division of the Physical Sciences. Moreover, the department recognizes that students enter applied mathematics from diverse backgrounds, and that some otherwise well qualified students may require more than one year to satisfy the requirements described below.

To obtain the degree of Master of Science in mathematics under the auspices of CAMP, the candidate must meet the departmental requirements stated above, with the modification that the nine graduate courses to be passed are not restricted to those listed above. These nine courses must, however, include the analysis sequence:

MATH Analysis I-II-III 300
31200-31300-31400

They must also include a second, approved three quarter sequence of mathematics courses. This will normally be a sequence of applied mathematics courses emphasizing differential equations, ordinary and partial, and their numerical treatment. They may, however, consist of the algebra or topology sequence.

A third approved sequence of courses may be chosen from the offerings of the Department of Mathematics or from those of another department. Possible choices of sequences outside the Department of Mathematics are:

Astronomy & Astrophysics
ASTR 30100 Stars 100
ASTR 30200 Astrophysics-2 100
ASTR 30300 Interstellar Matter 100

Chemistry
CHEM 36100 Wave Mechanics and Spectroscopy 100
CHEM 36200 Quantum Mechanics 100
CHEM 36300 Statistical Thermodynamics 100

Economics
ECON 30500 Game Theory 100
ECON 30600  THE ECONOMICS OF INFORMATION  100
ECON 30700  Decision Theory  100

Geophysical Sciences
GEOS 35100  Fndls Of Fluid Mechanics  100
GEOS 35200  Geophysical Fluid Dynamics  100
GEOS 35300  Dynamics of Viscous Fluids  100

Physics
PHYS 32200  Advanced Electrodynamics I  100
PHYS 32300  Advanced Electrodynamics II  100

and a third course to be approved

The requirements for the Ph.D. in applied mathematics are the same as the departmental requirements listed above.

MATHEMATICS COURSES

MATH 30200-30300. Computability Theory I-II.
The courses in this sequence are offered in alternate years.

MATH 30200. Computability Theory I. 100 Units.
CMSC 38000 is concerned with recursive (computable) functions and sets generated by an algorithm (recursively enumerable sets). Topics include various mathematical models for computations (e.g., Turing machines and Kleene schemata, enumeration and s-m-n theorems, the recursion theorem, classification of unsolvable problems, priority methods for the construction of recursively enumerable sets and degrees).
Instructor(s): R. Soare Terms Offered: Winter
Prerequisite(s): Consent of department counselor. MATH 25500 or consent of instructor.
Equivalent Course(s): CMSC 38000

MATH 30300. Computability Theory II. 100 Units.
CMSC 38100 treats classification of sets by the degree of information they encode, algebraic structure and degrees of recursively enumerable sets, advanced priority methods, and generalized recursion theory.
Instructor(s): R. Soare Terms Offered: Winter, Spring
Prerequisite(s): Consent of department counselor. MATH 25500 or consent of instructor.
Equivalent Course(s): CMSC 38100
MATH 30300. Computability Theory II. 100 Units.
CMSC 38100 treats classification of sets by the degree of information they encode, algebraic structure and degrees of recursively enumerable sets, advanced priority methods, and generalized recursion theory.
Instructor(s): R. Soare Terms Offered: Winter, Spring
Prerequisite(s): Consent of department counselor. MATH 25500 or consent of instructor.
Equivalent Course(s): CMSC 38100

MATH 30500. Computability and Complexity Theory. 100 Units.
Part one of this course consists of models for defining computable functions: primitive recursive functions, (general) recursive functions, and Turing machines; the Church-Turing Thesis; unsolvable problems; diagonalization; and properties of computably enumerable sets. Part two of this course deals with Kolmogorov (resource bounded) complexity: the quantity of information in individual objects. Part three of this course covers functions computable with time and space bounds of the Turing machine: polynomial time computability, the classes P and NP, NP-complete problems, polynomial time hierarchy, and P-space complete problems.
Instructor(s): A. Razborov Terms Offered: Winter
Prerequisite(s): Consent of department counselor and instructor
Equivalent Course(s): CMSC 38500

MATH 30900-31000. Model Theory I-II.
MATH 30900 covers completeness and compactness; elimination of quantifiers; omission of types; elementary chains and homogeneous models; two cardinal theorems by Vaught, Chang, and Keisler; categories and functors; inverse systems of compact Hausdorff spaces; and applications of model theory to algebra. In MATH 31000, we study saturated models; categoricity in power; the Cantor-Bendixson and Morley derivatives; the Morley theorem and the Baldwin-Lachlan theorem on categoricity; rank in model theory; uniqueness of prime models and existence of saturated models; indiscernibles; ultraproducts; and differential fields of characteristic zero.

MATH 30900. Model Theory I. 100 Units.
MATH 30900 covers completeness and compactness; elimination of quantifiers; omission of types; elementary chains and homogeneous models; two cardinal theorems by Vaught, Chang, and Keisler; categories and functors; inverse systems of compact Hausdorff spaces; and applications of model theory to algebra.
Prerequisite(s): MATH 25500 or 25800
Note(s): This course is offered in alternate years.
MATH 31000. Model Theory II. 100 Units.
MATH 31000 covers saturated models; categoricity in power; the Cantor-Bendixson and Morley derivatives; the Morley theorem and the Baldwin-Lachlan theorem on categoricity; rank in model theory; uniqueness of prime models and existence of saturated models; indiscernibles; ultraproducts; and differential fields of characteristic zero.
Terms Offered: Spring
Prerequisite(s): MATH 30900
Note(s): This course is offered in alternate years.

MATH 31000. Model Theory II. 100 Units.
MATH 31000 covers saturated models; categoricity in power; the Cantor-Bendixson and Morley derivatives; the Morley theorem and the Baldwin-Lachlan theorem on categoricity; rank in model theory; uniqueness of prime models and existence of saturated models; indiscernibles; ultraproducts; and differential fields of characteristic zero.
Terms Offered: Spring
Prerequisite(s): MATH 30900
Note(s): This course is offered in alternate years.

MATH 31200-31300-31400. Analysis I-II-III.
Analysis I-II-III

MATH 31200. Analysis I. 100 Units.
Topics include: Measure theory and Lebesgue integration, harmonic functions on the disk and the upper half plane, Hardy spaces, conjugate harmonic functions, Introduction to probability theory, sums of independent variables, weak and strong law of large numbers, central limit theorem, Brownian motion, relation with harmonic functions, conditional expectation, martingales, ergodic theorem, and other aspects of measure theory in dynamics systems, geometric measure theory, Hausdorff measure.
Terms Offered: Autumn
Prerequisite(s): MATH 26200, 27000, 27200, and 27400; and consent of director or co-director of undergraduate studies

MATH 31300. Analysis II. 100 Units.
Topics include: Hilbert spaces, projections, bounded and compact operators, spectral theorem for compact selfadjoint operators, unbounded selfadjoint operators, Cayley transform, Banach spaces, Schauder bases, Hahn-Banach theorem and its geometric meaning, uniform boundedness principle, open mapping theorem, Frechet spaces, applications to elliptic partial differential equations, Fredholm alternative.
Terms Offered: Winter
Prerequisite(s): MATH 31200
MATH 31400. Analysis III. 100 Units.
Topics include: Basic complex analysis, Cauchy theorem in the homological formulation, residues, meromorphic functions, Mittag-Leffler theorem, Gamma and Zeta functions, analytic continuation, monodromy theorem, the concept of a Riemann surface, meromorphic differentials, divisors, Riemann-Roch theorem, compact Riemann surfaces, uniformization theorem, Green functions, hyperbolic surfaces, covering spaces, quotients.
Terms Offered: Spring
Prerequisite(s): MATH 31300

MATH 31300. Analysis II. 100 Units.
Topics include: Hilbert spaces, projections, bounded and compact operators, spectral theorem for compact selfadjoint operators, unbounded selfadjoint operators, Cayley transform, Banach spaces, Schauder bases, Hahn-Banach theorem and its geometric meaning, uniform boundedness principle, open mapping theorem, Frechet spaces, applications to elliptic partial differential equations, Fredholm alternative.
Terms Offered: Winter
Prerequisite(s): MATH 31200

MATH 31400. Analysis III. 100 Units.
Topics include: Basic complex analysis, Cauchy theorem in the homological formulation, residues, meromorphic functions, Mittag-Leffler theorem, Gamma and Zeta functions, analytic continuation, monodromy theorem, the concept of a Riemann surface, meromorphic differentials, divisors, Riemann-Roch theorem, compact Riemann surfaces, uniformization theorem, Green functions, hyperbolic surfaces, covering spaces, quotients.
Terms Offered: Spring
Prerequisite(s): MATH 31300

MATH 31700-31800-31900. Topology and Geometry I-II-III.

MATH 31700. Topology and Geometry I. 100 Units.
Topics include: Fundamental group, covering space theory and Van Kampen’s theorem (with a discussion of free and amalgamated products of groups), homology theory (singular, simplicial, cellular), cohomology theory, Mayer-Vietoris, cup products, Poincare Duality, Lefschetz fixed-point theorem, some homological algebra (including the Kunneth and universal coefficient theorems), higher homotopy groups, Whitehead’s theorem, exact sequence of a fibration, obstruction theory, Hurewicz isomorphism theorem.
Terms Offered: Autumn
Prerequisite(s): MATH 26200, 27000, 27200, and 27400; and consent of director or co-director of undergraduate studies
MATH 31800. Topology and Geometry II. 100 Units.
Topics include: Definition of manifolds, tangent and cotangent bundles, vector bundles. Inverse and implicit function theorems. Sard’s theorem and the Whitney embedding theorem. Degree of maps. Vector fields and flows, transversality, and intersection theory. Frobenius’ theorem, differential forms and the associated formalism of pullback, wedge product, integration, etc. Cohomology via differential forms, and the de Rham theorem. Further topics may include: compact Lie groups and their representations, Morse theory, cobordism, and differentiable structures on the sphere.
Terms Offered: Winter
Prerequisite(s): MATH 31700

MATH 31900. Topology and Geometry III. 100 Units.
Topics include: Riemannian metrics, connections and curvature on vector bundles, the Levi-Civita connection, and the multiple interpretations of curvature. Geodesics and the associated variational formalism (formulas for the 1st and 2nd variation of length), the exponential map, completeness, and the influence of curvature on the topological structure of a manifold (positive versus negative curvature). Lie groups. The Chern-Weil description of characteristic classes, the Gauss-Bonnet theorem, and possibly the Hodge Theorem.
Terms Offered: Winter
Prerequisite(s): MATH 31800
MATH 32500. Algebra I. 100 Units.
Topics include: Representation theory of finite groups, including symmetric
groups and finite groups of Lie type; group rings; Schur functors; induced
representations and Frobenius reciprocity; representation theory of Lie groups
and Lie algebras, highest weight theory, Schur-Weyl duality; applications of
representation theory in various parts of mathematics.
Terms Offered: Autumn
Prerequisite(s): MATH 25700-25800-25900, and consent of director or co-director
of undergraduate studies

MATH 32600. Algebra II. 100 Units.
This course will explain the dictionary between commutative algebra and
algebraic geometry. Topics will include the following. Commutative ring
theory; Noetherian property; Hilbert Basis Theorem; localization and local
rings; etc. Algebraic geometry: affine and projective varieties, ring of regular
functions, local rings at points, function fields, dimension theory, curves,
higher-dimensional varieties.
Terms Offered: Winter
Prerequisite(s): MATH 32500

MATH 32700. Algebra III. 100 Units.
According to the inclinations of the instructor, this course may cover: algebraic
number theory; homological algebra; further topics in algebraic geometry and/
or representation theory.
Terms Offered: Spring
Prerequisite(s): MATH 32600

MATH 32600. Algebra II. 100 Units.
This course will explain the dictionary between commutative algebra and algebraic
geometry. Topics will include the following. Commutative ring theory; Noetherian
property; Hilbert Basis Theorem; localization and local rings; etc. Algebraic
group theory; Schur functors; induced representations and Frobenius reciprocity; representation theory of Lie groups
and Lie algebras, highest weight theory, Schur-Weyl duality; applications of
representation theory in various parts of mathematics.
Terms Offered: Autumn
Prerequisite(s): MATH 25700-25800-25900, and consent of director or co-director
of undergraduate studies

MATH 32700. Algebra III. 100 Units.
According to the inclinations of the instructor, this course may cover: algebraic
number theory; homological algebra; further topics in algebraic geometry and/or
representation theory.
Terms Offered: Spring
Prerequisite(s): MATH 32600
MATH 37500. Algorithms in Finite Groups. 100 Units.
We consider the asymptotic complexity of some of the basic problems of computational group theory. The course demonstrates the relevance of a mix of mathematical techniques, ranging from combinatorial ideas, the elements of probability theory, and elementary group theory, to the theories of rapidly mixing Markov chains, applications of simply stated consequences of the Classification of Finite Simple Groups (CFSG), and, occasionally, detailed information about finite simple groups. No programming problems are assigned.
Instructor(s): L. Babai Terms Offered: Spring
Prerequisite(s): Consent of department counselor. Linear algebra, finite fields, and a first course in group theory (Jordan-Holder and Sylow theorems) required; prior knowledge of algorithms not required
Note(s): This course is offered in alternate years.
Equivalent Course(s): CMSC 36500

MATH 37760. Modern Signal Processing. 100 Units.
This course covers contemporary developments from time-frequency transforms and wavelets (1980s) to compressed sensing (2000s), a period during which signal processing significantly evolved and broadened to become the "mathematics of information". Topics: Review of classical sampling theory: Shannon-Nyquist, aliasing, filtering. Time-frequency transforms. Frame theory. Wavelet bases and filterbanks. Sparsity and nonlinear approximation. Algorithms: basis pursuit and matching pursuit. Compressed sensing. Matrix completion. Special topics: curvelets, phase retrieval, superresolution. Students who already have an interest in medical imaging (MRI, CT), or geophysical data processing (seismic, e-m), for instance, are welcome. The course assumes some affinity with undergraduate mathematics. The evaluation will consist of homework problems, and a project of the student’s choice. The project can either consist in reproducing results from the literature, or can be research-oriented.
Instructor(s): L. Demanet Terms Offered: Autumn
Prerequisite(s): Linear algebra and multivariate calculus
Equivalent Course(s): STAT 37760

MATH 38300. Numerical Solutions to Partial Differential Equations. 100 Units.
This course covers the basic mathematical theory behind numerical solution of partial differential equations. We investigate the convergence properties of finite element, finite difference and other discretization methods for solving partial differential equations, introducing Sobolev spaces and polynomial approximation theory. We emphasize error estimators, adaptivity, and optimal-order solvers for linear systems arising from PDEs. Special topics include PDEs of fluid mechanics, max-norm error estimates, and Banach-space operator-interpolation techniques.
Instructor(s): L. R. Scott Terms Offered: Spring. This course is offered in alternate years.
Prerequisite(s): Consent of department counselor and instructor
Equivalent Course(s): CMSC 38300
MATH 38509. Brownian Motion and Stochastic Calculus. 100 Units.
A rigorous introduction to Brownian motion and the stochastic integral.
Terms Offered: Autumn
Prerequisite(s): STAT 38300 or MATH 31200, or permission of the instructor.
Equivalent Course(s): STAT 38500

MATH 38511. Brownian Motion and Stochastic Calculus. 100 Units.
This is a rigorous introduction to the mathematical theory of Brownian motion and
the corresponding integration theory (stochastic integration). This is material that
all analysis graduate students should learn at some point whether or not they are
immediately planning to use probabilistic techniques. It is also a natural course for
more advanced math students who want to broaden their mathematical education
and to increase their marketability for nonacademic positions. In particular, it is
one of the most fundamental mathematical tools used in financial mathematics
(although we will not discuss finance in this course). This course differs from the
more applied STAT 39000 in that concepts are developed precisely and rigorously.
Instructor(s): G. Lawler
Terms Offered: Autumn
Prerequisite(s): The usual prerequisites are either the first-year graduate
mathematical analysis sequence (mainly the material in MATH 31200) or STAT
38100-38300, the first two quarters of the statistics measure-theoretic probability
sequence.
Equivalent Course(s): STAT 38510

MATH 38815. Geometric Complexity. 100 Units.
This course provides a basic introduction to geometric complexity theory, an
approach to the P vs. NP and related problems through algebraic geometry and
representation theory. No background in algebraic geometry or representation
theory will be assumed.
Instructor(s): K. Mulmuley
Terms Offered: Autumn. This course is offered in alternate years.
Prerequisite(s): Consent of department counselor and instructor
Note(s): Background in algebraic geometry or representation theory not required
Equivalent Course(s): CMSC 38815